ELSEVIER

PII: S0309-1708(97)00030-4

Advances in Water Resources Vol. 21, No. 8, pp. 691–696, 1998

0309-1708/98/$19.00+0.00

Nonmonotonic moisture profile as a solution of Richards' equation for soils with conductivity hysteresis

A.R. Kacimov* & N.D. Yakimov

Kazan University, Institute of Mathematics and Mechanics, University Street 17, 420008 Kazan, Russia

(Received 15 October 1996; accepted 10 July 1997)

Qualitative analysis of similar moisture profiles advancing in dry soils during the constant rate infiltration is proposed. The Richards partial derivative equation is reduced to the ordinary one, which is easily integrated. Using simple kinematic relations nonmonotonic saturation–depth profiles travelling both in 1D case and in 2D finger-type flows are interpreted as a consequence of hysteresis of conductivity depending on water content. The estimations made allow the description of fingering as a result of hysteresis. The mean velocity of hysteretic finger-type flow can be higher than velocity of 1D hysteretic flow while the last advances faster than the usual monotonic front without hysteresis. © 1998 Elsevier Science Limited. All rights reserved

Key words: infiltration, fingering, hysteresis, moisture, conductivity.

1 INTRODUCTION

Unsaturated flow in the subsurface, nonuniform in time and space, is of interest in hydrology, soil science, irrigation and drainage practices.[8,9,13] Many causes can induce this non-uniformity: matrix heterogeneity, multiphase nature of the flow and varying intensity of external moisture sources, among others. Recent experiments and analysis[5,6,11,16] showed that for both 1D and 2,3D cases moisture (capillary pressure) at fixed points within homogeneous layers exhibits pronounced maxima with time. In other words, during constant intensity infiltration along soil surface, tensiometers register in an initially dry matrix, saturation variations of travelling wave type. Intuitively, these water 'plugs' can be imagined as punching the 'conductance' barrier of the dry soil. Fingers or non-trivial flat fronts propagating in soils lead to important practical consequences discussed in details[5,6,16] (see also the references in these papers).

Numerical procedures based on finite difference and finite element methods treat effectively saturated–unsaturated seepage in heterogeneous soils. However, for preliminary estimations and elucidation of basic mechanisms governing the flow, simple analytical solutions are still of interest. In this note, we support analytically the idea[5,6] that hysteresis is the main mechanism of the time moisture maxima in homogeneous layers exposed to infiltration. It allows for description of the process in terms of the Richards' equation involving hysteretic conductivity–moisture curve. In contrast with the traditional assumption on nonhysteretic nature of this curve,[10] we derive fingering just from this hysteresis mechanism. There is much evidence confirming the existence and even the significance of the conductivity–moisture hysteresis.[3,4,9,18] For simplicity, we do not take into account the 'main' hysteresis in suction pressure–moisture. We deem that experiments[16] can be explained considering just the conductivity–moisture hysteresis.

We construct a solution of the Richards' equation that propagates as a 'rigid' profile. This solution analysed for a nonhysteretic case[3] is an example of the kinematic wave solution and exclusion of a free surface. Note that reduction of the partial differential equation for unsaturated flow to ordinary ones (similar solutions) is a traditional analytical method (see Bear,[2] pp. 498–500). The assumptions made below, especially about the wetting–drying curves, are very strong and the model used is rather artificial. However, it makes it possible to elucidate qualitative features of the

*Corresponding author.